ON THE PROCEDURE FOR THE DETERMINATION OF THE PROBABILITY OF COLLISION DAMAGE

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The present SOLAS regulations on probabilistic damage stability have shown anomalies for irregular or complex compartment arrangements. In this paper it is motivated that the source of those problems can be found in the use of a limited set of crisp, sharp, damage boundaries, which arose from the analytical integration of the underlying Probability Density Functions. A possible remedy would be to postpone the integration process until an actual compartment or set of compartments is evaluated. For practical reasons numerical integration is the most applicable method for that task. This proposal is elaborated and implemented in an experimental computer program, which is used to test the approach on a number of example cases. These tests have shown that there is a numerical deviation between conventional SOLAS and the proposed approach, which comes as no surprise. For practical reasons it is investigated whether new density functions, derived with the conventional method, in combination with numerical integration could lead to numerical compatibility. Finally, the effects of the current work on revision of the SOLAS rules are discussed, recommendations are made and subjects for further research are identified.

1. Introduction

Since 1992 probabilistic damage stability regulations for dry cargo ships are in effect, and since that time a lot of practical experience was gained. It appeared that in many cases the probabilistic method works as expected, but also cases of theoretically invalid behaviour have been experienced. Although the literature on this particular subject is scarce, the author has summarized a few aspects in [7] and [8]. In [14] calculations for different designs, with different degrees of subdivision and produced with different computer programs have been compared. The differences between several software packages were in the range from ≈ 0 to ≈ 0.05 on A. It also appeared

that in one case a finer subdivided vessel had a 7% lower A than a coarser subdivided vessel, for which there is no obvious explanation.
Although a robust and reproducible probabilistic damage stability method is a goal on its own, the tendency of using optimization methods is an extra motive to strive for a better calculation procedure, which must also be *automated* in the context of optimization. Such optimization methods try to achieve an optimal subdivision, applied optimization methods can be conventional, genetic algorithm (e.g. [1] and [11]) or Response Surface Model ([22]).
The quest for a better calculation procedure forms the main content of this paper, but first the experienced problems with the conventional method will be discussed shortly.

2. Problems with the conventional calculation procedure

A detailed discussion of the encountered problems with the method of [19] and the accompanying explanatory notes [4] is presented in [8]. Summarized the different aspects are:

1. Only one damage per compartment is taken into account. A compartment with multiple branches must be split virtually into different parts.
2. A non-regular layout, as shown e.g. in Figure 1 is not processable. The probability of damage of such a configuration can only be calculated after a virtual subdivision into smaller, rectangular, parts. Such a further division into fictitious compartments, as e.g. proposed by [6], can make a non-regular configuration regular and therefore processable, although it also introduces a number of disadvantages:

   - Not the intended damage case is calculated, but something else. In the practice this aspect might be of minor importance, and the calculation results might be valid, but on the main point it is something else.
   - The subdivision method is not universal, will it be subdivided with transverse or with longitudinal virtual bulkheads or a combination of both? Although the calculation results might be equal, it can be annoying when different parties use a different subdivision strategy for the same vessel.
   - The number of damage cases may increase drastically, if all existing non-regular compartments have to be subdivided. Because several classification societies have proven to have major problems with above-average numbers of damage cases, this fact will not speed up the approval process.

3. Compartments bounded by one ore more warped bulkheads cannot be processed. An example of such a configuration is given in Figure 2.

Figure 1. Non-regular compartment lay-out.

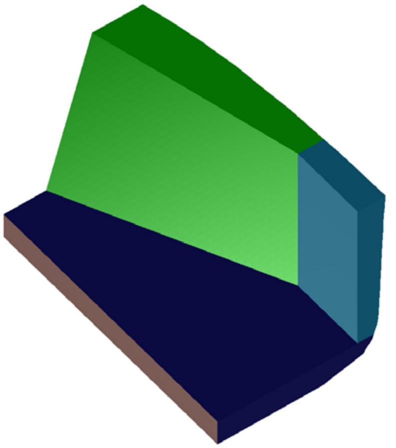

Figure 2. Warped bulkhead.

4. The SOLAS system may intrinsically lead to negative probabilities, as demonstrated in [5] with an example, represented here in Figure 3, which was presented to the IMO-SLF subcommittee in document SLF 37/5/2 in the early nineties.

Figure 3. Barge with narrow side compartments.

5. Incompatibilities between calculations produced by different computer programs. In the Netherlands a group of shipyards and classification societies has been investigating the divergence in results, based on the plan as proposed in [20]. At the moment of writing the working group has not drawn conclusions yet, but tentatively we can mention some sources of incompatibilities:

   - Incompatible subdivision. Although the SOLAS rules denote the 'compartment' as the basic subdivision entity, a zonal subdivision is frequently adopted instead, whereas a 'zone' is generally used as a longitudinal interval between some boundaries which are chosen on the basis of human

heuristics. Comparing a calculations based on compartments on the one hand, and 'zones' on the other is quite difficult, because a different set of damage cases will be used.
- The explanatory notes state quite clearly in part III that the penetration depth *b* shall be measured from the shell. Nevertheless it appeared that some well-known computer programs measure this distance from B/2 instead. This discrepancy makes comparison between different calculations difficult.
- Computer output is often not complete, has an unfamiliar nomenclature, or relies on unknown assumptions. This hampers a quick comparison between calculations produced by different software.

These incompatibilities cause that a ship designer can never be certain that a design which complies according to the design calculations will in the end also be accepted by the classification society or national authority. The reason is that these bodies do not evaluate an issued calculation as such, but make their own independent calculation. If the results differ it is Sisyphean labour to find the source of the differences, and in most cases the approving bodies simply assume that their own calculation is correct, or that the calculation with the lowest A is representative. This mechanism makes the ship design process rather unpredictable.

It must be emphasized that the mentioned problems occur with the conventional SOLAS method. At this moment IMO has agreed upon updated formulations (see Section 8) which may dissolve some of these problems. On the other hand, until the moment that new regulations come into effect the world has to live with the old ones, so we continue with the focus on SOLAS.

3. The origin of problems

The first three aspects as discussed in the previous section are caused by the fact that the conventional method assumes that a side damage, which extents from baseline upwards, is bounded by four boundaries, namely two transverse boundaries (aft and forward), one longitudinal boundary (the inside boundary, which is not necessarily parallel to centerplane) and one horizontal boundary (the upper boundary). Firstly, the number of *four* is simply too small for modelling a complex damage case. Secondly, the location of those boundaries is not always evident, this is the issue of semantics, which is discussed in §3.3 of [8] into more detail. The fourth aspect of the previous section, the negative probabilities, is caused by the fact that in the equation of the probability of a two-compartment damage $pr = p_{12}.r_{12} - p_1.r_1 - p_2.r_2$ ([4] Figure A-3) r_1 and/or $r_2 \gg r_{12}$. This subject is also addressed in Sub-section 7.2.

4. A remedy

In the previous section we have motivated that a major source of the problems lies in the application of damage boundaries as such; a damage is assumed to be of a simple shape (i.e. rectangular or trapezoidal) and assumed to have a limited number of crisp boundaries. We could avoid the troubles if this crisp-boundary concept could vanish. In order to investigate this possibility we will take a quick tour into statistics. The common approach to model stochastic events is 1) record the events, 2) construct a histogram on the basis of these recordings, 3) normalize this histogram, and approximate it with a Probability Density Function (PDF) and 4) integrate this PDF in order to obtain a Cumulative Distribution Function (CDF). This CDF can be evaluated conveniently to predict the occurrence of events below or above a certain magnitude. This approach was, and is, also used in the context of probabilistic damage stability for ships.
If we traverse these steps it becomes obvious that the crisp-boundary concept is not an intrinsic property of the events which are modelled. It is in the fourth step, the integration, where the crisp boundaries are introduced, after all the CDF is the *definite* integral of the PDF. Now that the source is identified, the solution is rather simple:

> Do not use a priori definite integration, but postpone the integration until an actual damage case is evaluated.

Given the possible complexity of actual compartment geometries analytical integration will in general not be feasible, so numerical integration must be applied instead. Such a transition from analytical integration to numerical integration is not uncommon in the history of naval architecture, take for instance the calculation of hydrostatic properties of a ship hull. Within living memory numerical integration is used for this task, because a hull shape is in general too complex to be modelled analytically. A more recent analogy can be found in the determination of strength and stiffness properties; when a construction becomes too complex to be assessed analytically, numerical (Finite Element) methods are applied.
So the proposed method is not revolutionary, on the contrary, it is becoming common practice that a direct calculation variant can be used besides rule-driven legislation.

5. Further elaboration of the proposal

Before the integration procedure can be elaborated, we first have to identify the PDF's in the different directions. This work is done in the next three sub-sections.

5.1. *Transverse subdivision only: $f(\overline{x},\overline{y})$*

There is some confusion over the question whether the two random variables $\overline{x}$ and $\overline{y}$ are independent. It is advocated in [12] and [13] that since the domain of these two variables is a triangle, they cannot be independent. On the other hand both [19] and proposals for new regulations (see Section 8) declare the two variables independent, on basis of an analysis of the damage statistics. Because we concentrate on the [19] implementation, the latter approach is followed, so the joint probability density function $f(\overline{x},\overline{y})$ can be written as the product of two functions a and b:

$$f(\overline{x},\overline{y}) = a(\overline{x})b(\overline{y}). \tag{1}$$

The PDF's of a and b were not derived in [19] nor in [4], but it was shown in e.g. [5] that

$$a(\overline{x}) = \begin{cases} 0.4 + 1.6\overline{x} & \text{if} \quad \overline{x} < \frac{1}{2} \\ 1.2 & \text{if} \quad \overline{x} \geq \frac{1}{2} \end{cases} \tag{2}$$

and

$$b(\overline{y}) = \begin{cases} \frac{2}{J_{max}} \quad \left(1 - \frac{\overline{y}}{J_{max}}\right) & \text{if} \quad \overline{y} < J_{max} \\ 0 & \text{if} \quad \overline{y} \geq J_{max} \end{cases} \tag{3}$$

where both $\overline{x}$ and $\overline{y}$ are defined in the interval [0,1]. In order to limit the damage within the vessels boundaries there are two additional requirements: $\overline{x} - \overline{y}/2 \geq 0$ and $\overline{x} + \overline{y}/2 \leq 1$. However, if the damage is to be limited within these boundaries the total probability of damage will not be unity but some other figure P_{tot}, for instance 0.9341 for $J_{max} = 0.24$ as demonstrated by [5]. This undesirable effect can be avoided in three ways:

1. Modify the PDF, e.g. by replacing Eq. 1 by $f(\overline{x},\overline{y}) = a(\overline{x})b(\overline{y})/P_{tot}$.

2. Allow the damage to extend beyond the vessel's boundaries.

3. Use a special treatment for the forward and aft compartments. In SOLAS this method is used.

Numerically the last two methods are equivalent. Because the second method requires the least programming effort, we have chosen that one for our implementation.

5.2. *Longitudinal subdivision: $c(\overline{z},\overline{y})$*

The presence of longitudinal subdivision is in [19] taken into account by means of the 'reduction factor r'. If this r is a CDF the corresponding PDF can be obtained by differentiation, but neither in [19], nor in [4] this r is derived, it is not even called a 'probability'. Also in [3], where a similar r equation is applied, it is systematically called 'reduction factor'. Nevertheless the r-factor has the *character* of a probability, which is shown by the fact that the product pr is called a 'probability' in the regulations. Also the fact that r tends to unity when the penetration approaches centerplane gives it the nature of a CDF.
In §IV.3.e of the explanatory notes of [3] (which is copied into §3.5 of [4]) it is stated that r is a conditional distribution of $\overline{z}$ given $\overline{y}$, so we can derive the corresponding PDF $c(\overline{z},\overline{y})$ by differentiating $r = r(\overline{z}|\overline{y})$ to $\overline{z}$ only, as worked out in Eq. 4:

$$
\begin{array}{lllll}
\text{For } \overline{y} \geq \overline{z}/5 & & & & \\
\quad \text{if } \overline{z} \leq 0.20 & c(\overline{z},\overline{y}) = \dfrac{\partial r}{\partial \overline{z}} & = & 2.3 + \dfrac{0.08}{\overline{y}+0.02} & \\
\quad \text{if } \overline{z} > 0.20 & c(\overline{z},\overline{y}) = \dfrac{\partial r}{\partial \overline{z}} & = & 1 & \\
\text{For } \overline{y} < \overline{z}/5 & & & & (4) \\
\quad \text{if } \overline{z} \leq 0.20 & c(\overline{z},\overline{y}) = \dfrac{\partial r}{\partial \overline{z}} & = & 4.5\dfrac{\overline{y}}{\overline{z}^2} - 2\dfrac{\overline{y}}{(\overline{z}+0.1)^2} & \\
\quad \text{if } \overline{z} > 0.20 & c(\overline{z},\overline{y}) = \dfrac{\partial r}{\partial \overline{z}} & = & \dfrac{\overline{y}}{\overline{z}}\left\{\dfrac{3.2}{\overline{z}} - \dfrac{0.4}{\overline{z}+0.1}\left(\dfrac{1}{\overline{z}} + \dfrac{1}{\overline{z}+0.1}\right)\right\} &
\end{array}
$$

When using this PDF for numerical integration one should make arrangements to model the infinities of r at $\overline{z} = 0$ and $\overline{z} = \frac{1}{2}$ properly. In Figure 4 this PDF is drawn, on the left side including the infinities, while on the right side these have been left out for better visualization.

5.3. *Horizontal subdivision: $w(h)$*

The derivation of the PDF $w(h)$ of the v-factor of [19] is rather straightforward. v can be differentiated to h, as elaborated in Eq. 5.

$$
\text{if} \quad h < d \quad \text{or} \quad h > H_{max} \quad \text{then} \quad w(h) = \frac{\mathrm{d}v}{\mathrm{d}h} = 0 \quad \text{else} \quad w(h) = \frac{\mathrm{d}v}{\mathrm{d}h} = \frac{1}{H_{max} - d} \qquad (5)
$$

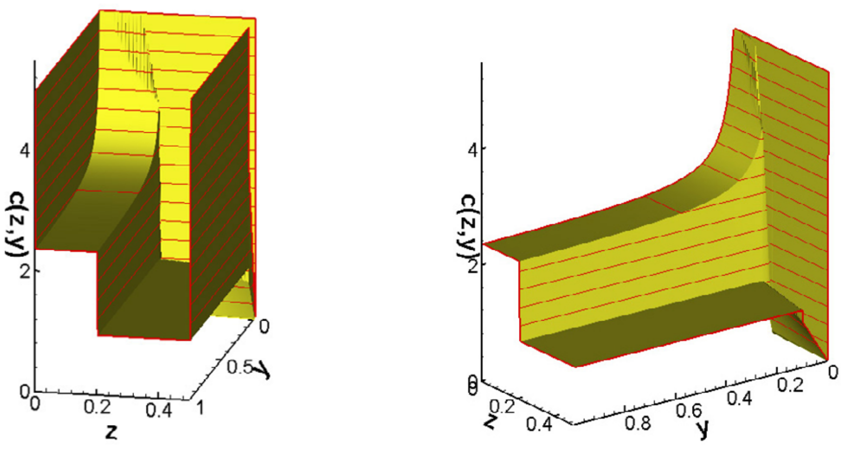

Figure 4. Probability density function $c(\overline{z},\overline{y}) = \frac{\partial r}{\partial \overline{z}}$ *(cut off at* $c(\overline{z},\overline{y}) = 5$*).*

5.4. Integration procedure

The probability of damage to a single compartment *prv* is

$$prv = \int_{d1}\int_{d2}\int_{d3}\int_{d4} \quad f(\overline{x},\overline{y}) \quad c(\overline{z},\overline{y}) \quad w(h) \quad \mathrm{d}\overline{x}\mathrm{d}\overline{y}\mathrm{d}\overline{z}\mathrm{d}h, \tag{6}$$

where the integration domains $d1..d4$ are chosen in such way that any combination of $\overline{x}$, $\overline{y}$, $\overline{z}$ and h only damages the compartment under consideration. With linear numerical integration Eq. 6 is replaced by

$$prv = \sum_{d1}\sum_{d2}\sum_{d3}\sum_{d4} \quad f(\overline{x},\overline{y}) \quad c(\overline{z},\overline{y}) \quad w(h) \quad \Delta\overline{x}\Delta\overline{y}\Delta\overline{z}\Delta h. \tag{7}$$

The summation steps $\Delta\overline{x}$, $\Delta\overline{y}$, $\Delta\overline{z}$ and Δh can be chosen freely, there is no need for a uniform step. When applied to a single compartment, in our implementation at least one step on each internal bulkhead or deck is used, and an additional number of equidistant steps is added in order to improve the accuracy. In each summation step in Eq. 7 it is checked whether the combination of $\overline{x}$, $\overline{y}$, $\overline{z}$ and h actually only involves the single compartment under consideration (taking into account that the rupture is a rectangular side damage, which extents from baseline upwards). If this condition is not met, that particular summation step does not contribute to *prv*. A multi-compartment damage is treated in the same manner; first integration steps are chosen over the entire extent of all compartments involved, and for each summation step it is checked whether all the intended compartments, but not more than the intended compartments, are affected. Only in that case this particular step contributes to *prv*. Please observe that, contrary

to the conventional method, a multi-compartment damage does not require any subtraction, so, assuming that all involved PDF's are non-negative over their entire range, negative *prv* values are impossible.
We have discussed damages to a single compartment and to multiple compartments, but vessels with a shaped stern or stem might also experience a zero-compartment damage. This effect is illustrated in Figure 5, where damage in the shaded areas fall within the limits of the rectangular box between the vessel's extremities, and can as a consequence be assigned a probability. Because there is no actual damage the probability of survival is 1.00 (assuming that the survival probability of the intact vessel is indeed unity). With the numerical integration method this effect can indeed be observed.

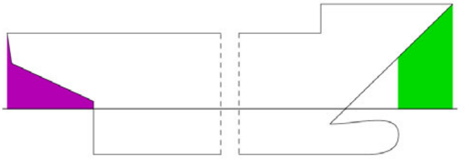

Figure 5. The shaded areas do also contribute to the probability of damage.

The number of additional summation steps can be chosen freely by the program user, so he can determine the trade-off between calculation time and accuracy in a way which suits him in a particular situation. Of course with a high number of summation steps the calculation time will increase, but that is the price which has to be paid for the increased accuracy. And one must not forget that also a conventional calculation can take a considerable amount of time, not only for the damage stability calculations, but also for the determination of the damage boundaries of each damage case. The latter task may be quite time-consuming for a complex compartment configuration, even if this task is performed by a dedicated numerical optimization method.
Finally, it can be remarked that the integration procedure is not limited to application over the entire vessel, but can also be used to determine probabilities of damage to parts only. This capacity can be used to calculate 'local subdivision indices', as e.g. proposed by [21].

6. Application examples

The numerical integration procedure, as elaborated in the previous section, was implemented in an experimental test program. With this program a number of test cases has been calculated, the results of which are presented below. The first cases are for a schematic vessel and/or compartment configuration, while the last case concerns an actual vessel.

6.1. Application examples with schematic subdivision configurations

The first example is the barge of Figure 3, for which the application of SOLAS in some cases leads to negative probabilities. As shown in the second column of Table 1, the direct integration method does deliver only non-negative probabilities.

Table 1. Probabilities of damage of the barge of Figure 3.

Damage case	SOLAS method	Numerical integration	Simulated SOLAS
E	0.3539	0.3539	0.3539
A	0.0128	0.0196	0.0128
B	0.0401	0.0574	0.0401
C	0.0534	0.0719	0.0534
A&E	0.0228	0.0230	0.0175
A&D	0.0302	0.0235	0.0302
A&B	0.0184	0.0235	0.0184
B&D	0.1044	0.0870	0.1044
B&C	0.0215	0.0287	0.0215
C&D	0.1389	0.1205	0.1389
A&D&E	0.0522	0.0520	0.0575
A&B&E	0.0062	0.0054	0.0029
A&B&D	0.0581	0.0530	0.0581
A&B&C	-0.0006	0.0001	-0.0006
B&C&D	0.0738	0.0664	0.0739
A&B&D&E	0.0129	0.0137	0.0161
A&B&C&E	-0.0007	0.0000	-0.0007
A&B&C&D	0.0010	0.0003	0.0011
A&B&C&D&E	0.0007	0.0000	0.0007
Total	**1.0000**	**1.0000**	**1.0000**

See Sub-section 7.2 for a discussion of the last column

The second example concerns the non-regular layout of Figure 1. As mentioned in Section 2 the conventional SOLAS method cannot handle such a configuration as such, but with subdivision into fictitious compartments and a subsequent re-combination of the output the results can be obtained. Although there are infinite ways to choose the fictitious sub-compartments, the most obvious ones are subdivision by virtual longitudinal or by virtual transverse bulkheads. If virtual longitudinal bulkheads are used a total of 30 damage cases can be identified, and with virtual transverse bulkheads 40 damage cases exist. The numerical integration method, on the contrary, works on

Table 2. Probabilities of damage of the barge of Figure 1.

Damage case	SOLAS Longitudinal	SOLAS Transverse	Numerical integration
D	0.3846	0.3848	0.4057
C	0.2083	0.2083	0.2637
B&D	0.0241	0.0241	0.0311
A&D	0.0164	0.0164	0.0088
D&C	0.3193	0.3194	0.2643
A&B&D	0.0467	0.0467	0.0264
B&D&C	-0.0003	-0.0002	0.0000
A&D&C	0.0002	0.0001	0.0000
A&B&C&D	0.0004	0.0005	0.0000
Total	**1.0000**	**1.0000**	**1.0000**

the basis of the real compartment configuration, and only needs to process the 9 real damage cases. For the barge with dimensions 100 x 20 m the results are listed in Table 2. From this table we can conclude that according to the SOLAS method different fictitious sub-division strategies lead to identical results, at least for this example. We also see that the negative probabilities have vanished with the numerical integration method. However, the SOLAS results differ from those obtained by numerical integration, a subject which will further be addressed in Sub-section 7.2.

The next example case concerns a simple barge, with one warped longitudinal bulkhead. For this case no comparison with SOLAS results will be presented, because the only way to incorporate the warpness into a SOLAS calculation is a massive subdivision into tens or a few hundred of fictitious sub-compartments, which will lead to more than thousand damage cases. With the numerical integration method the integration step does have its effect on the results, and it is obvious that the results approximate their final value with an increasing number of steps. When using the same number of steps in $\overline{x}$, $\overline{y}$, $\overline{z}$ and h direction we found that with 20^4 steps the probability of damage of the side compartment was $98\frac{1}{2}\%$ of the real probability, with 50^4 steps $99\frac{1}{2}\%$ and with 75^4 steps 99.9%. Regardless of the step size, P is always unity.

Finally, an example is included to test the behaviour of the numerical integration method on horizontal subdivision. For that purpose a barge with a constant section shape according to Figure 7 of [8] was calculated. As far as the horizontal subdivision is concerned, no deviation from the conventional SOLAS results was found.

6.2. An application example with a realistic subdivision configuration

The proposed method was also applied to an actual general cargo vessel in the 100 m range. This vessel comprises 84 compartments, and 1920 damage cases can be identified for damages which involve up to and including 13 damaged compartments. This vessel was calculated by the conventional SOLAS method, as well as by numerical integration, the results are summarized in Table 3. With the numerical integration method the combined contributions of the damages which consist of more than 13 compartment amounts to 0.0096, bringing the total probability of damage to exactly unity. Table 3 shows that the cumulative probability of the numerical integration tends almost asymptotically to unity, while with SOLAS the total probabilities wiggles a bit in the neighbourhood of unity without ever reaching it exactly. By the way, for this case the numerical integration took less processing time than the conventional method.

Table 3. Contributions to probability of damage.

Number of damaged compartments	SOLAS	Cumulative SOLAS	Numerical integration	Cumulative Numerical
1	0.0206	0.0206	0.0146	0.0146
2	0.1228	0.1434	0.0429	0.0575
3	0.2339	0.3774	0.2076	0.2651
4	0.1371	0.5145	0.1862	0.4513
5	0.1322	0.6467	0.1377	0.5890
7	0.1706	0.8173	0.1582	0.7472
7	0.0800	0.8974	0.1242	0.8715
8	0.0598	0.9572	0.0495	0.9210
9	0.0824	1.0396	0.0249	0.9459
10	-0.0063	1.0333	0.0168	0.9627
11	0.0448	1.0782	0.0104	0.9731
12	0.0629	1.1411	0.0105	0.9836
13	0.0205	1.1616	0.0068	0.9904

7. Compatibility with the present SOLAS requirements

The subject of this section is to investigate the compatibility of the proposed numerical integration method with the conventional SOLAS method. Beforehand it can be doubted whether equal results will be obtained, after all one of our objections to the conventional method was the occurrence of negative probabilities. With the numerical integration method we have got rid of them, but that implies also that all results will be fundamentally different. Nevertheless we have two reasons to examine the compatibility issue further. The first reason is a practical one; for application in the daily ship design practice it could be a practical advantage if numerical compatibility could be achieved. The second reason is curiosity.

7.1. Transverse subdivision only: probability p

In Sub-section 5.1 the treatment at the vessel's extremities was chosen to be compatible with SOLAS. That means that in case of transverse subdivision full compatibility has been achieved, which is also indicated by the first damage case of Table 1, which is bounded by transverse bulkheads only, and which shows equal results for the SOLAS method and the numerical integration method.

7.2. Transverse and longitudinal: the r-factor

We have seen in the Tables 1, 2 and 3 that in case of combined transverse and longitudinal subdivision the results between conventional SOLAS and the numerical integration method diverge. That is no surprise, because the numerical integration method is based upon the multiple integral of Eq. 6, while in SOLAS this function is decomposed into the incompatible equation

$$prv = p.r.v = \int\int f(\overline{x},\overline{y})\mathrm{d}\overline{x}\mathrm{d}\overline{y} \quad . \quad \int c(\overline{z}|\overline{y})\mathrm{d}\overline{z} \quad . \quad \int w(h)\mathrm{d}h. \tag{8}$$

In order to achieve full numerical compatibility between SOLAS and the numerical integration method we will have to derive the underlying PDF's from Eq. 8 and substitute them in Eq. 6. At this moment we see no possibilities for an analytical derivation, but it could be worthwhile to try a numerical approach. For this purpose the conventional SOLAS method is treated as a black box, which is used to generate the p and r values of a densely subdivided rectangular barge. This numerical exercise is performed with the stand-alone 188-line Pascal computer program `solaspdf.pas`, which is available on Internet[1]. The probabilities which are generated in this way could be considered as samples of experiments with the black box, and could be used to generate new statistics, by first grouping results in histograms, and subsequently approximate them with new PDF's. Because our population is rather dense we skip those steps, and consider the samples as a numerical equivalent of such PDF's.

The SOLAS method provides an equation for probability p of a 2-compartment damage, $p = p_{12} - p_1 - p_2$, and also for pr, $pr = p_{12}.r_{12} - p_1.r_1 - p_2.r_2$, with similar equations for multi-compartment damages. For damage cases of more than one compartment there is no method to calculate r explicitly, but it can be obtained as the ratio

[1]This program, as well as color versions of the graphs, can be obtained from http://www.sarc.nl, 'Download' section, login as non-registered user, go to the probability_of_damage subdirectory.

of *pr* and *r*. Some results are shown in Figure 6, which show distributions of *pr* and *r* as a functions of $\overline{y}$ and $\overline{z}$.

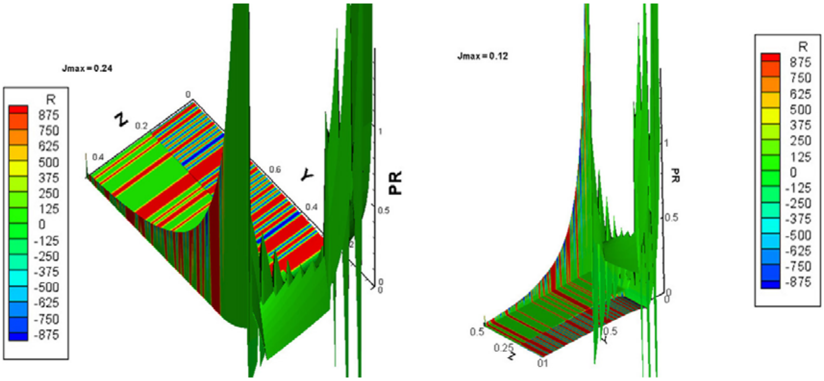

Figure 6. pr and r distributions according to SOLAS (graph available in color on Internet[1]*).*

One would expect reduction factor r to be in the interval [0,1], but instead the resulting r-values appear to have any value between $-\infty$ and ∞. The reason for this phenomenon is that in some cases $pr \neq 0$ while at the same time $p \approx 0$, so that $r = pr/p$ has a very large positive or negative value. With such large values of $|r|$ it is useless to draw r as function of $\overline{y}$ and $\overline{z}$, so in Figure 6 r is represented by a color distribution which is painted on the graph of pr, while, in order to avoid extreme values, $|r|$ is limited to 1000.

We consider the wild character of Figure 6 rather surprising, we had expected one as shown in Figure 7, which is the graphical representation of the product of Eqs. 1 and 4, instead. Maybe over the past 15 years people might have had the idea that the probabilistic damage stability was founded upon the piecewise smooth function of Figure 7, but it seems they have been working with the spiky field of Figure 6.

Anyway, we have used the data of Figure 6 as an alternative PDF, in the hope to achieve numerical SOLAS compatibility. For the barge of Figure 3 the obtained results are listed in the last column of Table 1. It is remarkable in this table that for many damage cases the probabilities of the conventional SOLAS method and the simulated SOLAS method are equal, except for damage case 'A&E' and cases which contain 'A&E'. The reason is that the conventional SOLAS calculation was made with the possibility of an angled inside damage boundary. For damage case 'A&E' the penetration at the aft side of the damage case can in that case be greater than at the forward side, so the mean penetration breadth is greater than the breadth of the wing tank. The simulated SOLAS calculation is performed by numerical integration, where all the integration steps are rectangular, so they have inside boundaries which are parallel to

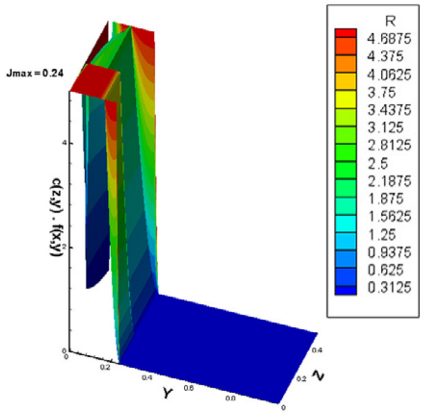

Figure 7. Underlying SOLAS PDF function $f(\overline{x},\overline{y})c(\overline{z},\overline{y})$ (cut off at $f(\overline{x},\overline{y})c(\overline{z},\overline{y}) = 5$).

center plane.

Except for this phenomenon the simulated SOLAS agrees with the conventional method. So for this simple configuration numerical compatibility can be achieved, but for a realistic vessel it might be expected that the waterline curvature will be the cause of differences. It will require additional investigations to verify whether the numerical integration method, based on the simulated SOLAS PDF's, can be applied in the daily ship design practice.

8. The future: revised SOLAS, and the HARDER project

Until sofar our attention was focused on the [19] regulations which are currently in force, and on the practical experience gained with these rules. However, at the moment of writing new insights and regulations in the field of probabilistic damage stability are under development. One development was the HARDER project, which was performed from 2000 to 2003. For the project description and the results we refer to [2] and [15]. HARDER contributed to multiple sub-subjects in the domain of probabilistic damage stability, but we will concentrate on the one which is relevant in our context: the probability of damage. Based on extended damage statistics and a new analysis, HARDER proposed updated density and distribution functions of damage location and damage dimensions, see [9] and [10]. In the context of this paper it is relevant that the new proposals appear to be theoretically more sound than the equations of [19], because the basis for the derivation of r is the multiple integral of Eq. 6, and not the SOLAS type of Eq. 8.

The recommendations of HARDER to the IMO are in the form of new p and r formulae, see e.g. [15] and [17]. By choosing this form HARDER sticks to the method-

ology of using CDF's for the calculation of the probability of damage, so it might be expected that the first three problems of Section 2 will persist. So, although *robustness* was an important issue in the HARDER project, due to the intrinsic problems of the applied conventional calculation procedure it was not fully achieved.
Currently, the SOLAS probabilistic damage stability regulations undergo a major revision, the status at the moment of writing can be found in [18]. The character of the p and r factors of the revised SOLAS are based on the HARDER proposals, albeit with modified J_{max} and some other factors. The formulae to determine p, r and v are given in the form of CDF's, but as a kind of background information also the PDF for p is included. If the revised SOLAS regulations would only be based on the conventional calculation procedure we have lost a chance for an improvement in terms of robustness and consistency. So it could be an option to prepare SOLAS also for the numerical integration method. For this purpose the following provisions could be considered:

- Include all relevant PDF's explicitly in the text.
- Mention the numerical integration method as a recognized alternative to the conventional method.
- Include a reservation on the definition of b. In Reg. 7-1 of [18] the rotation of the vertical plane which limits the damage penetration is limited by the ratio of the penetrations at the aft side and forward side, similar, but not identical, to the requirements of [4]. Such a provision is meaningless when numerical integration is applied, and it could prevent future confusion to explicitly limit its application to the conventional approach.
- Propose a standard for data exchange, for instance in new or revised explanatory notes. The purpose of this standard is to ease the verification process when calculations are submitted for approval.

From a regulatory point of view the numerical integration method has the advantage that it depends less on semantics, on notions and on words, than the conventional method. In this respect the warning of [16] is due here: *Words are vulnerable. So is legislation. Defects in the quality of legislation often appear only after the legislation has come into force*.

9. Conclusion and further research

We have proposed an alternative calculation procedure for the calculation of the probability of damage, based on numerical integration. It was argued, and illustrated with examples, that this proposal is more robust, flexible and reproducible than the conventional method. The problems of the conventional method for the calculation of the probability of damage concentrate on non-regular and warped compartment configurations, which can be handled integrally by the proposed method. Even a configuration

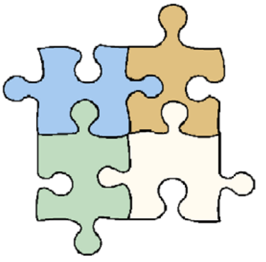

Figure 8. Curved compartment lay-out.

of curved compartments, as in Figure 8, poses no problem, which demonstrates the generality of the proposed method[2].

As a disadvantage of the proposed method can be mentioned the use of a large amount of damage cases, but that can also occur with a detailed conventional calculation. Besides, given the present hardware capacity a considerable number of damage cases ought not to be problematic. Another practical problem could be caused by the fact that there is no longer a one-to-one relationship between a damage case and a single compartment or a group of compartments; if calculations are issued for approval it will require comprehensive documentation and communication in order to give the approving body a clear picture of the calculation strategy. As areas of further research or attunement can be identified:

- Apply the proposed method on a wide range of actual vessel and compartment configurations, both with the native and the simulated SOLAS PDF's.
- Investigate the behaviour and details of the numerical integration method in combination with the revised SOLAS PDF's.
- Design a format for the exchange of data between different computer programs, in order to ease the verification process. As relevant data items one could think of damage cases, probability of survival for each damage case, summation steps and the contribution of each step to P and A.

[2]Ideally, legislation should cover the general case. However, the history of probabilistic damage stability rules shows that they are based upon specific, simple cases, such as rectangular barges and regular compartment layouts. More complex cases are not anticipated.

Nomenclature

A	=	Attained subdivision index
B	=	The greatest moulded breadth of the ship at or below the deepest subdivision loadline
b	=	Penetration depth
d	=	Draught
d1..d4	=	Domains of integration
h	=	Damage height
H_{max}	=	Maximum possible vertical extent of damage above the baseline
J_{max}	=	Maximum non-dimensional damage length
L_s	=	Subdivision length
p	=	Probability of damage of one compartment or a group of compartments, based on transverse subdivision only
P	=	Aggregated probability of damage = $\sum prv$
r	=	Reduction factor on p, taking into account the effect of longitudinal subdivision
v	=	Reduction factor on p, taking into account the effect of horizontal subdivision
x	=	Damage location
y	=	Damage length
$\overline{x}$	=	Non-dimensional damage location = $\frac{x}{\mathrm{L}_s}$
$\overline{y}$	=	Non-dimensional damage length = $\frac{y}{\mathrm{L}_s}$
$\overline{z}$	=	Non-dimensional damage penetration = $\frac{b}{\mathrm{B}}$
$a(\overline{x})$	=	Probability density function of damage location
$b(\overline{y})$	=	Probability density function of damage length
$c(\overline{z},\overline{y})$	=	Probability density function of damage penetration
$f(\overline{x},\overline{y})$	=	Joint probability density function of side damage
$w(h)$	=	Probability density function of damage height

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