HYBRID REPRESENTATION OF THE SHAPE OF SHIP HULLS

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Int. Shipbuild. Progr., **48**, no. 3 (2001) pp. 247-269

Received: June 2001
Accepted: June 2001

This paper proposes a new geometric modeling technique, called hybrid representation (H-rep), for modeling ship hulls. The novelty of H-rep is in the integration of wireframe, surface and solid representation in one common data structure, and in providing topological support to surface modeling as well as an enhanced technique for curve fairing. The primary goal is to achieve topological integrity and generality of the new model. After an introduction into geometric modeling, the authors discuss some problems of direct application of single-patch B-spline and NURBS surface representations in ship hull design. They claim that even if multi-patch extensions of these techniques are applied, the user may find difficulties in terms of the intuitive and straightforward externalisation of the form of ship hulls as well as of the exact modeling of the shape singularities. H-rep lends itself to a more intuitive and robust ship hull design methodology, and allows designers to follow their conventional line-oriented thinking in designing. The authors demonstrate the advantages of the hybrid representation by two examples of ship hull design.

1. Introduction

A determining factor in the appearance and performance of a ship is the hull form. It exerts its influence on many properties such as resistance, intact and damage stability, behaviour in a seaway, manoeuvrability, production costs, and aesthetic qualities of the vessel. The importance of the ship hull has already been aptly expressed: ‘Die

Entwicklung günstiger Schiffsformen ist die wichtigste Aufgabe der Schiffsbauwissenschaft‘ [1]. Development of the best form of a ship hull was a time-consuming and labour-intensive task in the past. Manual drawing of lines plan, manual lofting and construction of shell plate developments required not only time, but also expertise, and made ship hull design a cumbersome process. Nowadays, digital computers can provide effective support to the development and optimisation of variants. The applicability of the commercialised CAD systems, however, is often limited by the fact that the involved modeling techniques are general and, hence, do not consider the special requirements of ship hull design. It is not only an applicability issue, but also an efficiency one. This motivated the authors to develop a dedicated modeling technique, which allows achieving efficiency, flexibility and adaptability in ship hull design.
Rather than introducing a brand new approach, which might be theoretically supported but methodologically not harmonising with the daily practice of the ship hull designers, the authors realised their objectives by further developing and combining existing geometric modeling techniques. They stated their requirements mainly based on their experiences as ship designers as well as based on a systematic analysis of the multi-representational functions of the commercial CAD packages. They have developed a supporting theory and, based on it, a methodology for multi-purpose geometric modeling of ship hulls of complex morphologies. This paper mainly focuses on the offered surface modeling functionality. The second section gives a concise survey on the techniques for geometric modeling of free form shapes with local features. The third section introduces and explains the hybrid representation. In section four, the offered curve and/or surface fairing possibilities are discussed. Section five presents two examples for the application of the hybrid representation in ship hull design. Finally, section six concludes on the work and on the results.

2. A concise survey of modeling techniques for free form shape modeling

Geometric modeling of ship hulls creates a theoretically supported information structure, which captures metric and/or topological properties. Actually, various information structures can be used, depending on the purpose of modeling. Those information structures are the basis of representations, which can be characterised by their completeness. The distinction between complete and incomplete geometric representations is discussed in [2]. A complete geometric representation fully and uniquely portrays an object in terms of identification (names, identifiers), geometry (shapes, dimensions), topology (entities, connections), location (place, position) and attributes (material, colour). An incomplete representation does not capture each piece of information that is needed for the exhaustive portraying. The origin of incompleteness is the lack of potential of a given set of geometric modeling entities to reflect the observable geometric characteristics of the modeled objects sufficiently. Thus, completeness in a sense is a yardstick for the topological integrity and validity of a model. The nature of completeness lends itself to a classification of the various geometric representations (Figure 1) [3].

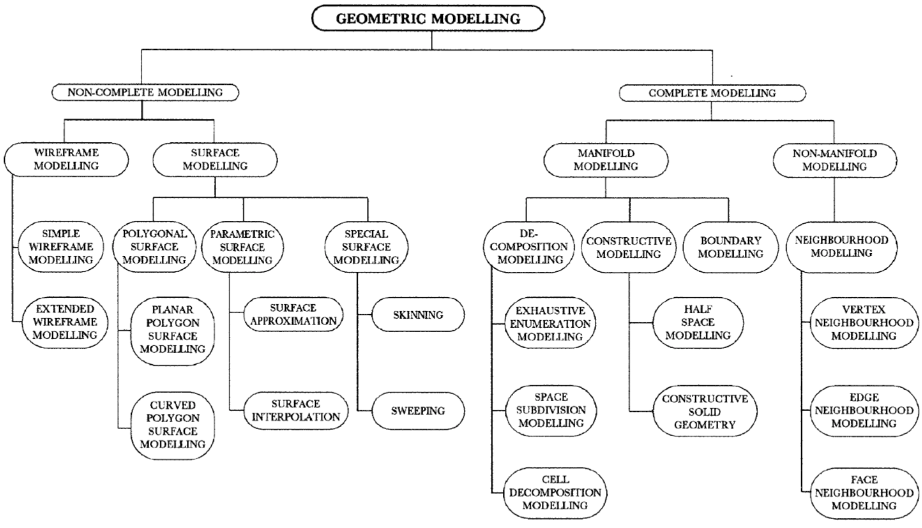

Figure 1. Classification of geometric modeling methods for rigid solids.

2.1. Conventional geometric modeling techniques for ship hull design

The information structures used for incomplete and complete geometric representations are well-documented [4], [5], [6]. Concentrating on the subject of geometric modeling of ship hulls, the various models and representations, as listed in Figure 2, are discussed in [7]. Several ship hull-modeling systems use simple wireframe representation, where the hull form is captured by a system of curves. The curves are rarely described explicitly, instead implicit or parametric representations are used. Recent systems typically use polynomial or spline-based representation of curves. The majority of the wireframe-based systems are used for engineering and analysis, rather than specifically for hull form design. Some modeling systems use wireframe coupled surface representation [8].

Surface representations offer the possibility of describing the free form ship hull with a system of surface patches. Early systems specialised for ship hull design often used Coons-patches [9]. However, in the last two decades parametric surface representations have become governing. The reason is that the general and specific implementations of Bézier, B-spline and/or NURBS techniques offer advantages that can hardly be provided by the formerly applied techniques. One of the most favourable properties of the B-spline/NURBS surfaces is that by manipulating the control points a great variety of sculptured objects can be produced, which can be processed and visualised very efficiently. Due to its strength in representing and visualising open hull surfaces, nearly all existing commercial hull form design systems have adopted the B-spline and/or NURBS surface representation. Due to the common mathematical basics, the functional differences between these systems are relatively small. The major difference can be found in the number of surface patches they use to model the ship hull. Some systems use single surface patch, while other systems use multiple surface patches with various fitting options. The major limitation of the existing B-spline and/or NURBS surface modeling systems is the lack of topological integrity and above all generality.

		GEOMETRIC MODEL				
		INCOMPLETE MODEL				COMPLETE MODEL
		SIMPLE WIREFRAME MODEL	EXTENDED WIREFRAME MODEL	POLYGONAL SURFACE MODEL	PARAMETRIC SURFACE MODEL	
GEOMETRY REPRESENTATION	NON-POLYNOMIAL	OBSOLETE				
	SINGLE POLYNOMIAL	FOR ANALYSIS ONLY				
	B-SPLINE	FOR ANALYSIS ONLY			DE FACTO STANDARD	
	NURBS				DE FACTO STANDARD	
	TRANSFINITE SURFACE		RESEARCH ONLY	FOR ANALYSIS ONLY		

Figure 2. Models and representations used in existing hullform systems.

2.2..Some problems with the conventional modeling techniques

Parametric B-splines and NURBS do offer a number of attractive properties for surface design such as local control, linear precision, the convex hull property and the variation diminishing property. Nevertheless, there are still open issues related to the application of advanced parametric surface modeling representations in ship hull design and some shortcomings have also been recognised. Actually these are not so much mathematical and computational issues, but pragmatic ones originating in the everyday practice of designers. For instance, these methods are very effective to represent a simple ship hull, but difficulties may arise in the support of intuitive designing of complex hull forms. Especially in the case of single patch shapes, B-splines and NURBS are challenged by abruptly changing compound shapes and local shape features at requested positions. An illustrative example is presented in [10]. Although by manipulating the control points a great variety of sculptured shapes can be produced, lengthy adjustments have to be made to arrive at a shape with the above-mentioned morphological properties.

The main issues relating to the application of B-splines/NURBS in the naval design can be summarised as follows: (a) constraints stemming in the rigidity of the network of control points, (b) limitations in representation of shape discontinuities of ship hulls, and (c) limited possibilities of direct interpolation of geometric points.

The primary issue related to parametric surface design is the rigidity of the network of control points. The standard formulae for the B-spline and NURBS representation imply a regular mesh. It means that the parameter directions of the mesh are orthogonal in the parameter space (Figure 3). A regular mesh goes together with the following noteworthy limitations:

- The defining curves of the regular mesh are in general not parallel to the main orthogonal planes of the vessel. Consequently, rather than working with curves of his/her own choice, the ship designer must be prepared to work with more or less arbitrary 3D curves over the surface of the ship hull.
- A regular mesh cannot include partial mesh curves. Partial mesh curves are desirable, for example, for partial waterlines, additional local shape details, additional local shape control and integrated stem rounding. If multiple regular surfaces are applied, the partial curves are supposed to end on a mesh curve. It produces so-called T-joints, which represent an irregular topology. Continuous transition between the neighbouring surfaces can be constructed only if the parameterisation on both sides of the T-joint is continuous. A problem appears when the parameterisation significantly deviates from the natural arc length parameterisation.
- Additional mesh curves may also be necessary for a precise definition of local shape details. However, if they run over the complete surface (to maintain regularity), they might cause undulations in the regions where they are superfluous. This effect is caused by the fact that in these regions simply too many mesh curves determine the shape of the ship hull. From the geometric modeling practice it is well known, that the fairest surface can be obtained with a minimum number of mesh curves.
- The curves in the regular mesh can typically be chosen by the designer with a proper parameterisation and can be interpolated. However, the mesh curves cannot be chosen arbitrarily in most of the used systems. It means the ship designer cannot choose exactly those curves, which deliver important shape characteristics. They have to be created by projection or intersection. Experiences show that the results of these actions are unpredictable.

- The location and nature of the mesh curves must be defined at the beginning of the surface design process. When a different set-up appears to be more appropriate later on, modification to a new arrangement is difficult. It is often quicker and easier to drop the work done so far and to start with a new arrangement.

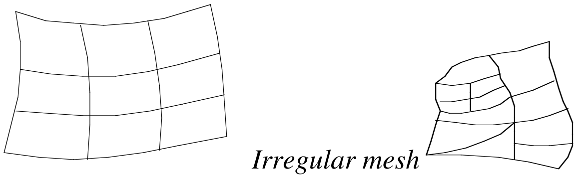

Figure 3. Mesh regularity.

As far as representation of shape discontinuities of a ship hull form is concerned, limitations originate from the fact that B-splines/NURBS have been developed for modeling GC^X continuous surfaces[i], and in practice X is limited to 2. Although it is possible to realise GC^1 discontinuities on a single surface patch by manipulating the control points, the knots, the weights or the spans, the place of them cannot be specified directly, and the tangency and curvature properties in the environment of the discontinuity are also difficult to control. In most of the customary ship hulls there are many discontinuities of zero order, first order or second order. For example over knuckles (1st order), or at the extremes of the midship section bilge or at a waterline rounding where the curvature is 2nd order discontinuous. In addition, there are cases in which the magnitude of curvature is suddenly changing and it results in a kind of near-discontinuity (for example at a bulbous bow). It typically occurs as an interaction between the global form of the ship hull and the sudden change in the magnitude of the curvature over a limited region.

The B-spline and likewise the NURBS representations offer the multiple knot point and line insertion technique to create discontinuities, and they have to be accounted for in a regular mesh context. However, it needs deep mathematical knowledge and widespread experiences. These are, in the overwhelming majority of cases, not readily available with industrial designers. The hottest problem of discontinuity representation emerges when discontinuity lines do not occur in a regular mesh. Thus, implementation of an intuitive, and practice-guided (rather than a theory-guided) approach for modeling specific features of hull forms can be problematical with one single B-spline or NURBS surface.

The exact representation of a simple midship section consisting of the three parts, called flat of bottom, bilge and flat of side in the naval engineering jargon (Figure 4), cannot be achieved easily with a single B-spline surface. The explanation is that it

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1 GC^X denotes geometrical continuity to the Xth order of differentiablity.

tends to smooth away the discontinuities at both sides of the bilge, regardless of the order and the number of knots of the spline. A NURBS surface can produce an exact cylindrical part [11], but the designer has to be aware of exact (mathematically requested) values for parameterisation, weight factors and control points. Anyway, it slows down the ship hull design process and requires specific knowledge and skills from the designers. These facts raise doubts about the intuitiveness and effectiveness of this approach. More importantly, the mesh might get irregular by the discontinuity. The mesh irregularity is obviously a topological issue, rather than a geometrical one. To avoid troubles with irregular surface regions is a task for an advanced ship hull modeling system.

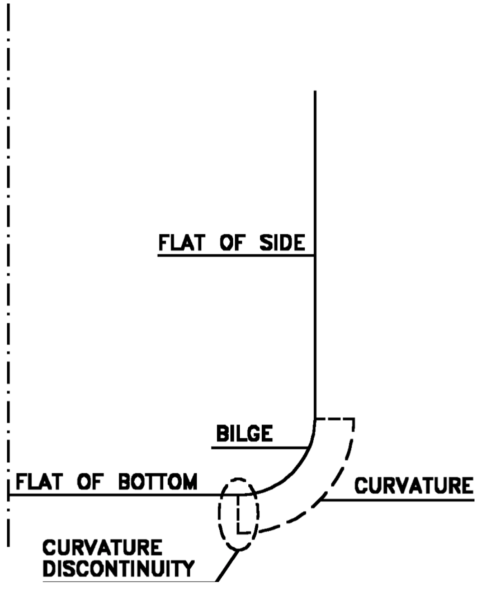

Figure 4. A typical midship section.

The third issue is the possibility of interpolation, that is, generating a compound surface through a set of known geometric points or curves. In the case of B-spline and NURBS representations it again cannot be done directly, but by means of various indirect techniques offered by the particular modeling system. Apart from the fact that it needs extra efforts, it limits the opportunities of ship hull form reconstruction that is based on digitised points or curves of existing hull forms. The designers can choose to go into iterative approximation or to start designing a completely new hull form, which overlays the known points or curves. From a computational point of view, the origin of the problem is in the knot value assignment of each individual point or curve to be interpolated.

To eliminate the problems discussed above, an obvious suggestion would be to use multiple surface patches for ship hull forms of complex morphology. Multi-patch-based B-spline or NURBS surface representation provides several advantages to single-patch-based surface design, but still three problems most probably occur:

- The system of surface patches is only geometrically arranged. Without a supporting topology checking the geometric model for closedness and validity is difficult.
- Definition and arrangement of the surface patches remain the responsibility of the ship designer. Only the past experiences can guide the designers to find the optimum selection and composition of the multiple patches.
- Taking care of the continuity and/or discontinuity conditions between the connected surface patches also remains a task for the designers. They need knowledge and experience to specify the constraints that play a role in the requested transitions.
- As a solution for the above-described problems we have incorporated a line-induced topological structure and a multi-patch surface generation facility with multiple parameter spaces.

3. Introducing hybrid representation for geometric modeling of ship hulls

The above analysis reveals that besides the advantageous features, parametric representations do have some less favourable features in practical applications. The application inconveniences are inherent, that is, they cannot be eliminated by smart programming or by a dexterous user-interface. Our investigation explored that by combining the wireframe representation with surface representation and a boundary-oriented solid representation, a specific information structure and representation scheme can be created that is very useful in ship hull design. It offers us a complete geometric model, which already proved its usefulness in other applications [12] [13]. Because this representation is based on multiple and heterogeneous data structures and involves multiple representation techniques, it is called hybrid representation and abbreviated as H-rep. It allows us to develop functions that are dedicated to ship hull design. Figure 5 shows the architecture of our modeling system. The wireframe representation is a subset of the extended boundary representation, hence it is integral both on data structure and on modeling methodology levels. In Figure 6 the H-rep data structure can be seen.

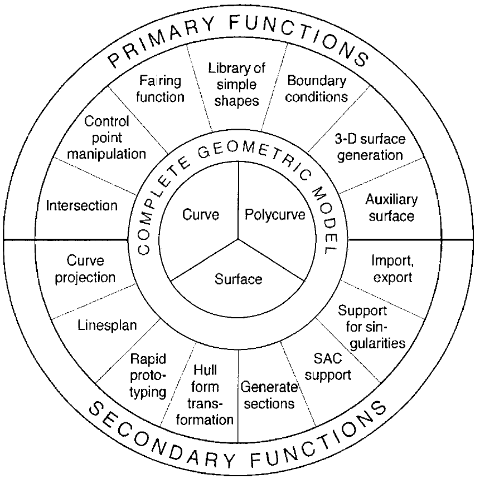

Figure 5. Functional scheme of the developed system.

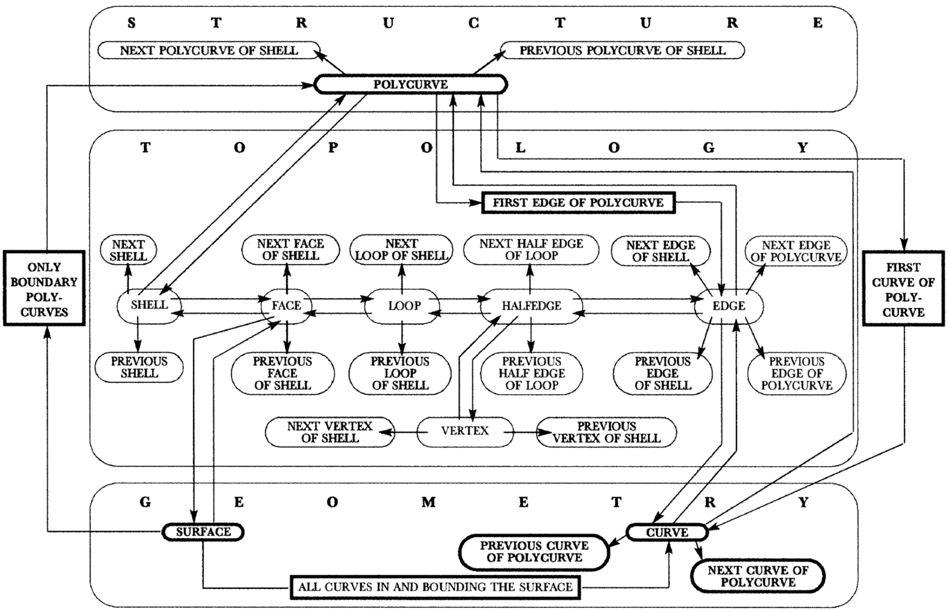

Figure 6. Data model of H-rep.

In addition to the above-mentioned geometrical modeling capabilities, a versatile ship hull design system is supposed to have other supporting functions. In order to make the daily use practical and convenient, a number of supporting functions has been implemented in our system. The system kernel incorporates various functions that can be used to generate and manipulate polycurves, including curve fairing techniques. There are functions dedicated to generation and visualisation of surface patches and patch complexes. Another group of functions facilitates the application of solid modeling operations on the model. These are called primary modeling functions. In addition, there are secondary functions that support the use of the system, such as linesplan generation and hullform transformation. Also an aid is included, backed by the Sectional Area Curve (SAC), which guides the designer goal-oriented towards a hullform with desired hullform coefficients, such as block coefficient and longitudinal centre of buoyancy,

Another group of auxiliary functions supports model import from other systems. It makes it possible to import files containing existing hull forms in the form of simple wireframe models and converting the wireframe models to H-rep models. A typical downstream function is the morphological segmentation of the ship hull for multi-axis milling and free form thick layer object manufacturing (FF-TLOM) [14]. In principle, the system works without these secondary functions, but in a less sophisticated way.

3.1. Geometric and topological entities and structures

The novelty in our approach is that it introduces a comprehensive topological structure to support the integration of wireframe, surface and solid modeling, and the conversion of an open surface model to a valid solid model. Some of the commercialised surface modeling systems suffer from the lack of an underlying general topological structure.

In our system, the simplest geometric entities are geometric points. Nevertheless, a linked sequence of curves, referred to as a polycurve, is considered to be the fundamental geometric entity. This entity makes it possible for the ship hull designers to work in the computer environment according to the conventional line-oriented design methodology. The polycurves are concerned both in the wireframe representation and in the surface representation. In our H-rep data structure, the edges are ordered not only around the faces (as it is with an ordinary B-rep), but over the complete hull surface. The structural arrangement of the polycurves is mapped to an edge-vertex topology that delivers a subset of the connectivity information for the extended boundary representation. Further modeling entities are surface patches. To enable effective surface design, multiple surface patches can be collected in multi-patches, i.e. into one surface. They are represented by an additional logical entity in the H-rep data structure.

The topology included in the H-rep follows the topological structure of the extended boundary representation and is implemented as a half-edge-based data structure [15]. From a programming point of view, the basis of the half-edge data structure is a set of relations of one half of an edge to other adjacent topological entities such as vertices, faces and other half-edges. Applied over the patch complexes, this topological structure makes it possible to convert the surface representation to solid representation. The boundary of the ship hull is topologically described by a shell entity. Parts of a shell, the face entities are the topological counterparts of finite and non-self intersecting surface patches. The boundary of a face consists of edges, and an ordered sequence of edges is mapped to an edge-loop. A face can be bounded by more than one loop entity. In such a case, one loop entity is designated to be the outer boundary and the other loop entities enclose gaps in the face. A non-self intersecting closed curve or a closed arrangement of curves is the geometric base of a loop. The above-mentioned half-edge entities are logical entities indicating the two possible orientations of a physical edge. The vertex entities are the boundaries of an edge and are the topological counterparts of metric points. The topological structure based on these entities enables the system to handle any arbitrary irregular mesh of surface patches.

3.2. Representation and manipulation of surface patches

3.2.1. Patch representation

In the related literature, the techniques for four-sided surface patch definition can be found, they generally revert to the work of Coons [9], Gregory [16] and Gordon [17]. Interested readers are referred to these works. For handling three-sided and N-sided patches (with $N > 4$) a number of techniques are summarised in [18]. In our system, the Gregory patch has been used as an implementation of four-sided surface patches, while for the representation of N-sided patches the method presented in [19] was adopted. In the case of a Gregory patch, the twist vectors $\mathbf{F}_{uv}$'s at the corners are replaced by a rational combination of $\partial \mathbf{F}_u(u,v) / \partial v$ and $\partial \mathbf{F}_v(u,v) / \partial u$.

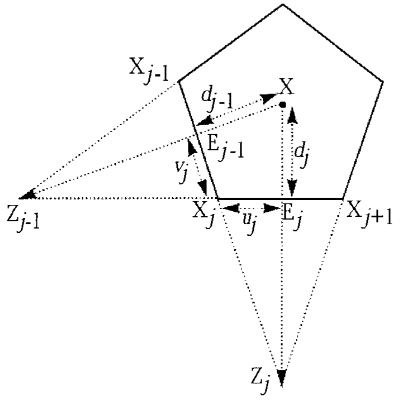

Figure 7. Coordinate map (in parameter space) of N-sided patch.

With the latter method an N-sided patch is represented as a weighted combination of N four-sided patches. The parameterisation can be uniform for each N-sided patch. As shown in Figure 7, a barycentric parameterisation is applied for each point **X** inside the boundary of the N-sided patch. The running index of the patch corners is j and ($j = 1 \ldots N$), The perpendicular distance of **X** to the side E_j is denoted by d_j. For each corner we have local parameters u_j and v_j, with:

$$u_j = d_{j-1} / (d_{j-1} + d_{j+1}) \tag{1a}$$

and

$$v_j = d_j / (d_{j-2} + d_j). \tag{1b}$$

Then, for each of the N sides, we have a positional function $\mathbf{f}_j(u)$, $j = 1 \ldots N$, and the cross-boundary derivative vector function $\mathbf{t}_j(u)$. For the j^{th} corner, we have two linear interpolants defined in terms of the local parameters u_j and v_j:

$$\mathbf{T}_1(u_j,v_j) = \mathbf{f}_{j-1}(v_j) + u_j\, \mathbf{t}_{j-1}(v_j) \tag{2a}$$

$$\mathbf{T}_2(u_j,v_j) = \mathbf{f}_j(u_j) + v_j\, \mathbf{t}_j(u_j) \tag{2b}$$

and a tensor product interpolant is:

$$\mathbf{T}_{12}(u_i, v_j) = \begin{bmatrix} 1 & u_j \end{bmatrix} \begin{bmatrix} \mathbf{f}_j(0) & \mathbf{t}_j(0) \\ \mathbf{t}_{j-1}(0) & \partial \mathbf{t}_j(0) / \partial u \end{bmatrix} \begin{bmatrix} 1 \\ v_j \end{bmatrix} \tag{3}$$

where $\partial \mathbf{t}_j(0)/ \partial u$ is the twist vector.
With these, the Boolean sum interpolant for the patch in each corner is:

$$\mathbf{p}_j(\mathbf{X}) = \mathbf{T}_1(\mathbf{X}) + \mathbf{T}_2(\mathbf{X}) - \mathbf{T}_{12}(\mathbf{X}) \tag{4}$$

The resultant interpolant is a weighted combination of the N corner patches:

$$\mathbf{p}(\mathbf{X}) = \sum_{j=1}^{N} w_j(\mathbf{X})\, \mathbf{p}_j(\mathbf{X}) \tag{5}$$

where $w_j(\mathbf{X})$ is a weight factor, and Π denotes repeated multiplication:

$$w_j(\mathbf{X}) = \frac{\prod_{i=1, i \neq j-1, i \neq j}^{N} d_i^2}{\sum_{k=1}^{N} \prod_{i=1, i \neq k-1, i \neq k}^{N} d_i^2} \tag{6}$$

3.2.2. Geometric continuity of surfaces between patches

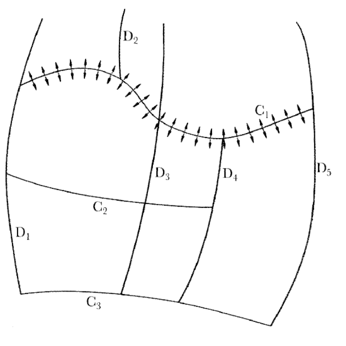

Figure 8. Patch layout and tangent ribbon.

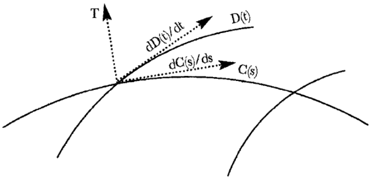

Figure 9. Construction of cross boundary derivative vector.

To ensure GC^1, i.e. tangent-plane continuity between adjacent patches, tangent ribbons must be constructed over the boundary curves of the surface patches. In the course of modeling the surface of the ship hull, the tangent ribbons supply tangency information for interfacing the patches. This is illustrated in Figure 8, which shows a mesh of curves C_i and D_j. As an example, the tangent ribbon of curve C_1 is computed by evaluating the connections with the curves $D_1 \dots D_5$. The tangent ribbons are constructed by means of a set of cross-boundary derivatives, which are calculated at each intersection between two curves. Of course, the cross-boundary derivative must be independent from the curve parameterisation. Therefore, as is suggested in [13] and [20], the Gram-Schmidt orthogonalization technique is applied. In Figure 9, a magnification of a part of the mesh of Figure 8 is sketched. It demonstrates how to construct the cross-boundary derivative vector **T** (which is co-planar with the tangent vectors d**D**(t)/dt and d**C**(s)/ds) with the tangents to the curves **C**(s) and **D**(t). The component of vector dD(t)/dt along dC(s)/ds is

$$\frac{\mathrm{d}\mathbf{D}(t)/\mathrm{d}t.d\mathbf{C}(s)/\mathrm{d}s}{\left|\mathrm{d}\mathbf{C}(s)/\mathrm{d}s\right|} \cdot \frac{\mathrm{d}\mathbf{C}(s)/\mathrm{d}s}{\left|\mathrm{d}\mathbf{C}(s)/\mathrm{d}s\right|} \tag{7}$$

so

$$\mathbf{T} = \mathrm{d}\mathbf{D}(t)/\mathrm{d}t - \frac{\mathrm{d}\mathbf{D}(t)/\mathrm{d}t\,.\,\mathrm{d}\mathbf{C}(s)/\mathrm{d}s}{\mathrm{d}\mathbf{C}(s)/\mathrm{d}s\,.\,\mathrm{d}\mathbf{C}(s)/\mathrm{d}s} \cdot \mathrm{d}\mathbf{C}(s)/\mathrm{d}s \tag{8}$$

The cross-boundary derivative vectors ***T*** are normalised, to make them independent from parameterisation. When programmed, this equation gives rather small cross-boundary derivative ***T*** for small angles between the tangents **C**(s)/ds and **D**(t)/dt. By the normalisation of ***T*** the possible approximation error is magnified, giving rise to undesired shape anomalies, further on in the surface generation process. Ignoring ***T***, when the angle between the tangents is smaller than a minimum value, could solve this problem. In our practical tests we observed that a minimum value of 15° gives satisfactory results.

3.2.3. Geometric continuity of curves between patches

The geometric continuity of curves between patches needs attention. So far we have considered the cases in which all curves on the surfaces meeting at a common node are compatible. If the surface curves are not compatible, i.e., the curves do not lie in one common tangent plane at their common node, first order discontinuities occur. To prevent this phenomenon, a scheme was proposed in [21] which modify the curves in order to ensure their compatibility. Because it would enhance our presented method, the inclusion of such a tangent-plane continuity correction scheme is considered.

4. Fairing of curves

The literature contains a variety of curve fairing methods, see [22] and [23] for an overview. Several of those methods matched to the polycurve and multi-patch manipulation concepts included in the H-rep, and would be suitable in our context. However, the basis of our curve fairing is the technique introduced by Dierckx [24]. Without going into much detail this method was chosen because it allows for local as well as global fairing. Furthermore it works on the basis of an intuitive geometric fairing criterion, which is the mean deviation between original and faired data points. The fairing is implemented in the following procedure. Let us start with:

- N given data points $\mathbf{q}_i$, with the corresponding weight factors w_i and parameter values t_i,
- allowed maximum mean deviation M, as specified by the designer, and
- the degree of the spline K = 3.

Our task is to find a smooth spline $\mathbf{f}_g(t)$ with g the number of knots, for which the sum of squared deviations is less than S, that is:

$$S = M^2 \sum_{i=1}^{N} w_i \tag{9}$$

We use fairness function J(f), which expresses the sum of the squares of the jump of the second order derivatives:

$$J(\mathbf{f}) = \sum_{i=2}^{g-1} (\mathbf{f}_g{}^{(k)}(t_i+) - \mathbf{f}_g{}^{(k)}(t_i-))^2, \quad \text{with } k=2. \tag{10}$$

Furthermore, we also need a function $E(\mathbf{f})$, which represents the closeness of fit and is defined as the sum of the weighted squared deviations:

$$E(\mathbf{f}) = \sum_{i=1}^{N} w_i \left| \mathbf{f}_g(t_i) - \mathbf{q}_i \right|^2 \tag{11}$$

To find the balance between fairness and closeness of fit, the function $J(\mathbf{f})$ must be minimized under the condition that $E(\mathbf{f}) \leq S$. This constrained minimization problem is solved by minimizing $J(\mathbf{f}) + pE(\mathbf{f})$, where p must be chosen so that $F_g(p) = E(\mathbf{f}) = S$. It was proved by Dierckx that when $p \to \infty$, $\mathbf{f}(t)$ tends to the Least Squares spline with g knots, and when $p = 0$, $\mathbf{f}(t)$ becomes the weighted Least Squares polynomial of degree K. Consequently, Dierckx's algorithm has two steps as a feature:

- find the minimum number of knots g as well as a knot distribution, for which $F_g(\infty) < S$, and
- find for this number of knots and for this knot distribution the value p^* for which $F_g(p) = S$.

To execute the first step, let us assume that we have knots $\lambda_1 \ldots \lambda_g$, and calculate the Least Squares spline $\mathbf{f}_g(t)$ determined by the given N data points $\mathbf{q}_i$. When writing out the equations we obtain an over-determined linear system of equations, which can be solved in several ways. Among other techniques the following two ones are applicable [25]:

- The most commonly used method of multiplying the observation matrix with its transpose, so that the normal equations are obtained, which can be solved with an ordinary matrix solver.
- Orthogonalization, where an upper triangular matrix is constructed, which can be solved easily by back substitution.

According to Dierckx, the matrix of the normal equations is banded as well as positive definite, so an efficient Cholesky solver for band matrices can be used. For large systems, however, the normal equations may become ill-conditioned that can cause numerical instability.

To maintain computational stability, the second method has been selected. Originally Dierckx applied the orthogonalization method together with the Givens-transformations, based on an alternative computation without square roots. In later implementations, conventional Givens-transformations with square roots have been used. The transformation with square roots is computationally less efficient, because of the relatively long evaluation of a square root. In order to make our computer system as efficient as possible, we have been experimenting with an implementation without square roots. No problems with the stability have been encountered.

When the spline $\mathbf{f}_g(t)$ has been determined on a knot sequence $\lambda_1 .. \lambda_g$, and $E(\mathbf{f})$ is still greater than S, a new knot λ must be added to the knot sequence, and a new iteration has to be executed to determine $\mathbf{f}_{g+1}(t)$. In order to decrease the number of iterations, Dierckx's implementation does take care of multiple simultaneous knot insertions. In our experiments there were cases with large number of inserted knots and rather asymmetrical distributions of the new knots. So in the H-rep implementation only one single knot is added at a time.

To determine the placement of the new knot, for all spans $\lambda_{j-1} .. \lambda_j$, $j = 2..g$, we calculate the errors δ_j:

$$\delta_j = \sum_{r=\mathrm{u}}^{v} w_r \left|\mathbf{f}_g(t_r) - \mathbf{q}_r\right|^2, \text{ with } \quad \lambda_{j-1} < t_\mathrm{u} < t_{\mathrm{u}+1} < t_{\mathrm{u}+2} .... < t_\mathrm{v} < \lambda_j \qquad (12)$$

The new knot is added in the parametric middle of the span j with the largest error δ_j. Once the number of knots g has been found for which $F_g(\infty) < S$, $F(0)$ is determined, and by means of an iterative numerical procedure the value p^* is found for which $F(p^*) = S$. The obtained control points of the spline are assigned a weight factor of unity, so actually a non-rational, non-uniform spline (NUBS) is generated.

5. Applications of hybrid representation in ship hull design

A complete discussion of the way of working with the new system goes beyond the extent of this paper. Our aim with the application examples is to illustrate for the reader the potential of and the opportunities offered by our approach of complex ship hull design. The theoretical backgrounds of geometric modeling are invisible to the ship designer. What he/she actually manipulates are familiar conventional entities such as curves, points and knuckles. The way of operation resembles the tradition of the hand-drawn lines plan. As a result, the system does not impose any limitation upon the nature of the hull form, but it is evident that the design of a complex ship always takes more time than the design of a simple one. To illustrate the possibilities, two actual ship designs are presented below. One demonstrates the reconstruction of a hull form, while the other one is an *ab initio* design.

5.1. Surface reconstruction without surface patches for a pollution control vessel

The first application example demonstrates the application of the extended wireframe representation in our system. The pollution control vessel "Neuwerk" was discussed in [26] and the body plan (of cross sections only) was published. The body plan of this vessel is rather complex, with many knuckles and discontinuities, which would make it rather cumbersome to model this vessel with a conventional parametric B-spline/NURBS surface. One reason is that the mesh of this vessel, consisting of cross sections and chines, is rather irregular, while another factor is the availability of only cross sections in the initial stage. Because the nature of the vessel does not impose any inherent parameterisation, the cross sections cannot be assigned a surface parameter value, so they cannot be used to generate an initial surface.

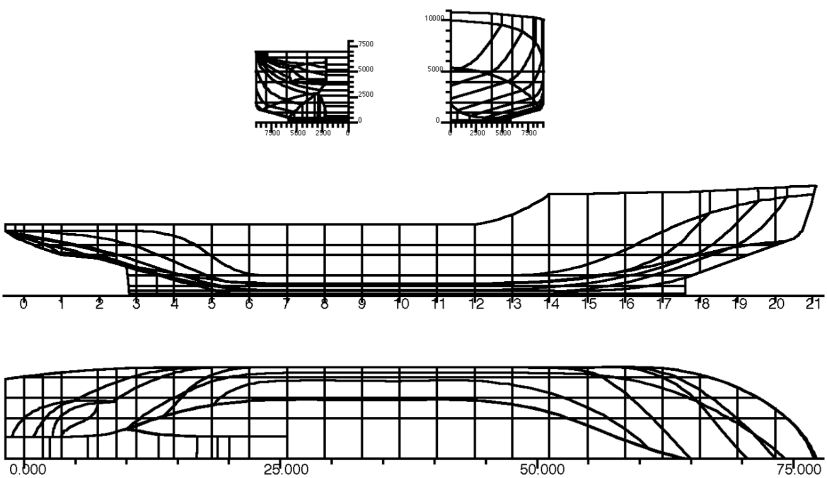

Figure 10. Pollution control vessel.

The first step in our reconstruction process was to digitise some cross sections, and to eliminate digitising errors by fairing them. Subsequently the chines were generated, simply by connecting the knuckles in each cross section. With this mesh of curves, additional waterlines and buttocks were generated and faired when it was necessary. For the reason that the initial mesh was quite dense, we did not use the surface patch representation capability to model the surfaces of the ship hull. The ship hull was reconstructed by interpolation on the basis of the available curves. The information carried by the wireframe model was sufficient for an enhanced visualisation of the hull. Figure 10 shows the resulting lines plan, while in Figure 11 the curved surfaces are shown.

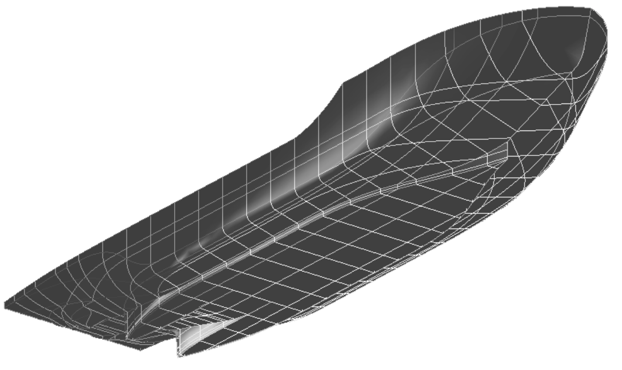

Figure 11. Rendered surfaces of the pollution control vessel.

5.2. Designing a frigate, with surface patches

To demonstrate the power of the surface capabilities, in this section we discuss the design of a frigate-like hull form. A fundamental consideration is that it is efficient for a human to do as little work as possible and let the computer take care of additional tasks, so the ship designer started with as little initial curves as possible, which still define the complete hull satisfactory. Initially a set of 11 curves, as shown in Figure 12, has been drawn interactively on the screen, with the aid of the NURBS curve control points. For the reason that each curve has its own parameterisation, the number of control points per curve may vary, so for each curve a minimum number of control points that were necessary to obtain the desired shape of that curve has been used. It made any additional curve fairing unnecessary. The choice to use exactly those 11 curves was entirely made by the designer, because there are no system-imposed directives in this respect. The only obligation for the designer is to let the curves start and end on other curves. Indeed the designer chose to include two curves which define the keel bar and the stem rounding, which are obviously details. It may seem premature to include details in an initial design stage, but when they are integrated from the very first start, they quite naturally arise in the design process, and do not require attention afterwards.

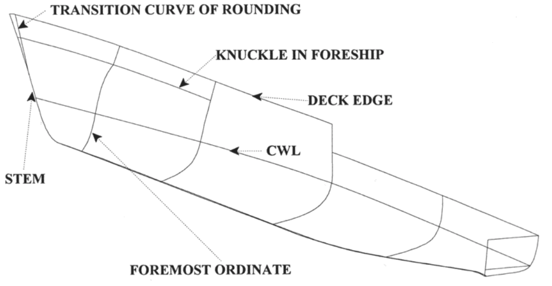

Figure 12. The initial set of 11 curves.

The number of edges bounding a patch, which determines whether the patch is three, four or N-sided, is resolved by the system, as well as the specific treatment of each patch type, with one of the methods discussed in paragraph 3.2.1. The T-joint which arises where the aft terminal of the fore ship knuckle meets the ordinate may seem a point of concern for the designer, because it introduces an irregularity in the mesh, but the patch continuity aspects are resolved entirely by the system, with the tangent ribbon method as discussed in paragraph 3.2.2.

Directly after the creation of the 11 initial curves the surface patches were generated. With the methods as discussed in paragraph 3.2.1 these patches derive their curved shape from the shape of the curves in the vicinity. A visual check of the resulting hull surface revealed two areas of undesired shape. The first concerned the cross sections near the transom, which were slightly too wide in the side parts, while the bilge area was too flat. The second showed undesired inflection points in the foremost ordinates. Whereas these are not *incorrect*, because they result from a valid surface interpolation, they are considered to be *undesired* from the designer's point of view. In order to avoid them, the set of curves was extended, which is shown in Figure 13. The first extension involved an additional ordinate in the aft ship, which forced the local shape into a more desired direction. Secondly, to give the sides of the ordinates near the transom more support, an additional partial waterline was generated in the aft ship, and modified to the desired shape. The last modification involved the fore ship undulation. It could have been countered with an additional waterline, midway between deck edge and construction waterline (CWL), but closer inspection of the set of curves, as shown in Figure 14, suggests another possibility. Because the transition curve of the rounding ends at the stem, somewhere between CWL and knuckle, this patch is bounded by CWL, stem, transition curve, knuckle and the foremost ordinate, so it is a five-sided patch with one rather small side. When the transition curve was connected at the intersection between CWL and stem a four-sided patch was created and the undulation disappeared. This example illustrates that although knowledge of the internal mechanisms of the user is not strictly necessary, because the system deals with the details automatically, the user may benefit from some background knowledge, because it may help in design decisions.

On the basis of the 13 curves of Figure 13 a set of curved surfaces, shown in Figure 15, is generated. Finally waterlines, buttocks and additional cross sections were projected upon these surface patches, resulting in the complete hull design of Figure 16.

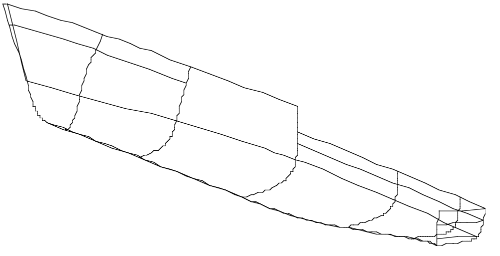

Figure 13. The extended set of 13 curves.

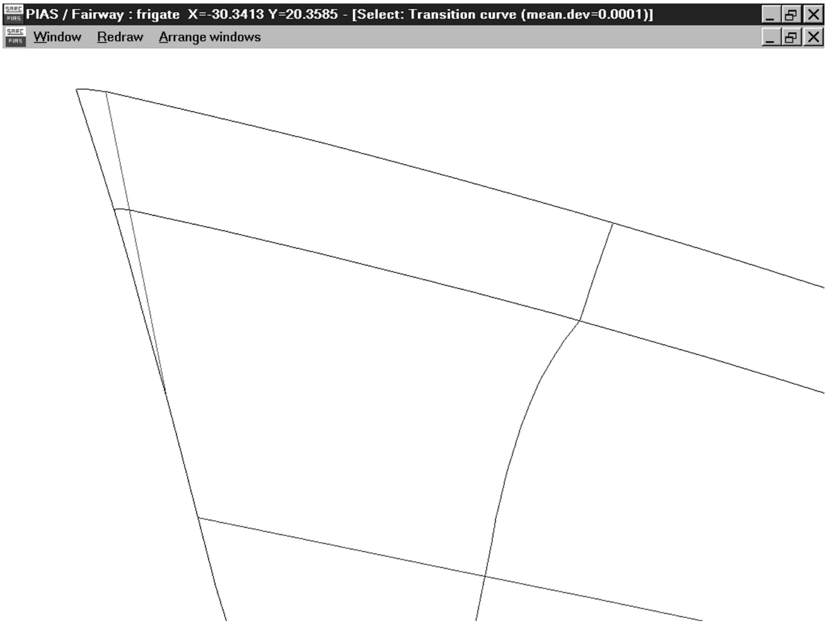

Figure 14. Magnification of the initial curve set.

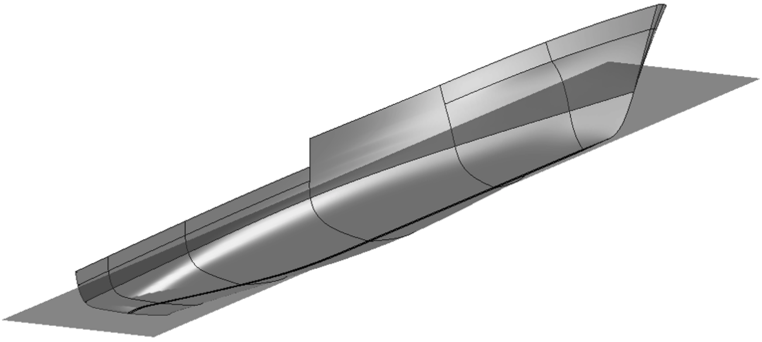

Figure 15. Curved surfaces on basis of the extended curve set, with also the CWL shown.

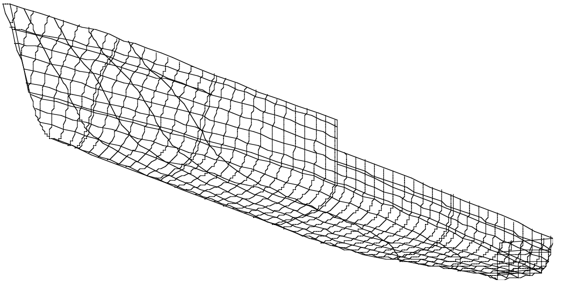

Figure 16. Additional waterlines, buttocks and cross sections, projected upon the hull surface.

6. Conclusions

The paper has described the results of an applied research in ship hull surface design. A hybrid representation technique for intuitive and efficient modeling of the shape of ship hulls and fairing of the surfaces of ship hulls has been presented. Compared with traditional methods, it provides a ship designer with more freedom and flexibility. The H-rep technique eliminates several restrictions upon the size or character of the shape of the ship hull to be designed. It does not imply any prescribed working sequence for the designer. Facilities for visual checking of the resultant shape are also provided by the geometric modeling package that has been developed based on the theoretical and methodological fundamentals. Applicability of the hybrid representation technique has been tested in the ship design practice. The advantages can be identified easily. Nevertheless, the modeling technique could benefit from a number of further enhancements, which could not be discussed in this paper, but will be considered in the follow up research.

Acknowledgement

The authors wish to express their appreciation to the anonymous reviewer of the paper, whose comments and recommendations were really useful in revising the paper and making it more transparent for the practitioners in naval architecture.

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